Divisibility Rules

What does "**divisible by**" mean?

"**Divisible by**" means when one integer is divided by another integer, the quotient is an integer. For example, given two integers *a* and *b*, if *a* is divisible by *b*, it means that when *a* is divided by *b*, the quotient is an integer and there is no remainder.
"a is **divisible by** b" also means "*b* is a **factor** of *a"* or "*a* is a **multiple** of *b".*

Divisibility Rules

A number is divisible by…
2: if the number is even or ends in 0, 2, 4, 6, or 8
3: if the sum of digits is divisible by 3
4: if the last two digits make a two-digit number that is divisible by 4
5: if the number ends in 0 or 5
6: if the number is divisible by 2 and 3
8: if the last three digits make a three-digit number that is divisible by 8
9: if the sum of digits is divisible by 9
10: if the number ends in 0
11: if the difference between the sum of the even-numbered digits and the sum of the odd-numbered digits is 0 or a multiple of 11

General Rule: The divisibility rule for an integer N, where $N = p \times q$, is if the number is divisible by p and q. (For example: The divisibility rule for the number 6)
Therefore:
12: if the number is divisible by 3 and 4 ($12 = 3 \times 4$)

Tip #1: A shortcut to find the remainder when a number is divided by 3 or 9 is to find the sum of the digits and divide it by 3 or 9.

Tip #2: A shortcut to find the remainder when a number is divided by 2, 5, or 10 is to find the last digit and divide it by 2, 5 or 10.

Tip #3: A shortcut to find the remainder when a number is divided by 4 is to find the two-digit number from the last two digits and divide it by 4.

Tip #4: A shortcut to find the remainder when a number is divided by 8 is to find the three-digit number from the last three digits and divide it by 8.

Example 1: Is 841 divisible by 3?

8 + 4 + 1 = 13
13 ÷ 3 = 4 R 1

Solution: We add the digits 8, 4, and 1 to test this. 8 + 4 + 1 = 13. 13 is not divisible by 3 so **841 is not divisible by 3**. Note: The remainder when 841 is divided by 3 is the same as when 13 is divided by 3 which is 1 (Tip #1!).

Answer: 841 is not divisible by 3

Example 2: Is 8,568 divisible by 9 and 8?

Solution: Let's add the digits to test if it's divisible by 9.
8 + 5 + 6 + 8 = 27. 27 is divisible by 9, so 8,568 is divisible by 9.

What about 8? Take the last three digits 568 and test for divisibility by 8.
568 ÷ 8 = 71 r 0

Answer: 8,568 is divisible by 8 and 9

Example 3: Which of these numbers {2, 3, 4, 5, 6, 8, 9, 10} is 180 divisible by?

Solution: 180 ends in 0 so it is divisible by 2, 5 and 10.
The sum of the digits is 1 + 8 + 0 = 9, so it is multiple of 3 and 9.
180 is a multiple of 6 because it is divisible by 2 and 3.
The last two digits is 80 and 80 is divisible by 4, so 180 is divisible by 4.
Finally, test for divisibility by 8. The last three digits of 180 are 180, so divide 8 from 180.
180 ÷ 8 = 22 r 4
Answer: {2, 3, 4, 5, 6, 9, 10}

Example 4: If the three-digit number 3A5 is divisible by 3, what is the sum of the possible values of A?

Solution: To test for divisibility by 3, add the digits.
$3 + A + 5 = 8 + A$ = multiple of 3, so 8 + A could be 9, 12, 15 but not 18 because that would make A = 10 and A has to be a single digit.

Set $\{8 + A = 9\}$, $\{8 + A = 12\}$, and $\{8 + A = 15\}$. The resulting values of A are 1, 4, and 7. The sum is 1 + 4 + 7 = **12.**

Answer: 12

Example 5: Is 142,637 divisible by 11?

Solution: According to the rule, add up the even-numbered digits and the odd-numbered digits. The even-numbered digits are 1, 2, 3 and the odd-numbered digits are 4, 6, 7.
1 + 2 + 3 = 6
4 + 6 + 7 = 17
17 - 6 = 11
The positive difference of these two numbers is 11 which means that **142,637 is divisible by 11**.

Answer: 142,637 is divisible by 11

Chapter 2

Review Problems

1. Which of these numbers are divisible by 5?   {365, 549, 766, 450, 324}

2. Is 108 divisible by 6?

3. How many whole numbers between 1 and 50 are divisible by 3?

4. Is 8,715 divisible by 15?   Hint: $15 = 5 \times 3$

5. What is the largest two-digit whole number that is divisible by 6?

6. What are the remainders when 123,456 is divided by 4, 5, 8, 9, or 10?

7. How many numbers between 1 and 100 are divisible by 7?

8. If the three-digit number 4A6 is divisible by 9, what is the value of A?

9. Which of these numbers is divisible by 8? {25,412, 34,612, 78,512, 56,388}

*10. The six-digit number 8A4,B57 is divisible by 11. What is the sum of A and B?

11. Jerry is thinking of a number between 1 and 100. The number is a multiple of 11 and is also a multiple of 5. What number is he thinking of?

*12. If 5,A3B is a multiple of 9, what is the sum of all possible values of A + B?

13. What is the remainder when 62,579,304 is divided by 9?

14. How many multiples of 3 are between 11 and 302?

15. The five-digit number A5,2A1 is divisible by 3. If the five-digit number is to be as large as possible, what is the value of A?

16. What is the smallest three-digit number that is a multiple of 5?

17. What is largest two-digit number that is divisible by 7 and 6?

18. Find the remainder when 7,649,432,789 is divided by 9.

19. Marina is trying to remember her favorite number. She remembers that the number is more than 40 and less than 70 and is divisible by 8 and 7. What is her favorite number?

*20. A number is randomly chosen from 1 to 100, inclusive. What is the probability that the number is divisible by 5 and 3? Express your answer as a percent.

Chapter 2 Solution Key

Review Problems

1. Which of these numbers are divisible by 5? {365, 549, 766, 450, 324}

Solution: The divisibility rule for 5 is if the number ends in 0 or 5.

Answer: 365 and 450

2. Is 108 divisible by 6?

Solution: The divisibility rule for 6 is if the number is divisible by 2 and 3.
108 is even, thus it is divisible by 2.
The sum of the digits is 1 + 0 + 8 = 9, which is divisible by 3.
Thus, 108 is divisible by 6.

Answer: Yes

3. How many whole numbers between 1 and 50 are divisible by 3?

Solution: 50 ÷ 3 = 16 R 2.
This means that the largest multiple of 3 less than 50 is 3 × 16 = 48.
There are 16 multiples of 3 between 1 and 50 (3 × 1, 3 × 2, …, 3 × 16).

Answer: 16 (numbers)

4. Is 8,715 divisible by 15? Hint: 15 = 5 × 3

Solution: Since 15 = 5 × 3, if 8,715 is divisible by 5 and 3, it is divisible by 15.
8,715 ends in 5 which means it is divisible by 5.
The sum of the digits is 8 + 7 + 1 + 5 = 21, which is divisible by 3.
Therefore, 8,715 is divisible by 15.

Answer: Yes

5. What is the largest two-digit whole number that is divisible by 6?

Solution: The largest two-digit whole number is 99.
99 ÷ 6 = 16 R 3.
Thus, 6 × 16 = 96 is the largest two-digit whole number that is divisible by 6.

Answer: 96

6. What are the remainders when 123,456 is divided by 4, 5, 8, 9, or 10?

Solution:
4: Last two digits: 56 ÷ 4 = 14 R **0**
5: Last digit: 6 ÷ 5 = 1 R **1**
8: Last three digits: 456 ÷ 8 = 57 R **0**
9: Sum of digits: 1 + 2 + 3 + 4 + 5 + 6 = 21; 21 ÷ 9 = 2 R **3**
10: Last digit: 6 ÷ 10 = 0 R **6**

Answer: 0, 1, 0, 3, 6

7. How many numbers between 1 and 100 are divisible by 7?

Solution: 100 ÷ 7 = 14 R 2.
This means that the largest multiple of 7 less than 100 is 7 × 14 = 98.
There are 14 multiples of 7 between 1 and 100 (7 × 1, 7 × 2, …, 7 × 14).

Answer: 14 (numbers)

8. If the three-digit number 4A6 is divisible by 9, what is the value of A?

Solution: Sum of digits: 4 + A + 6 = 10 + A. This value must be equal to a multiple of 9.
10 + A = 9; A = –1 10 + A = 18; A = 8 10 + A = 27; A = 17
Since A is a digit (0–9), the only value that works is 8. A must be 8.

Answer: 8

9. Which of these numbers is divisible by 8? {25,412, 34,612, 78,512, 56,388}

Solution: Only the last three digits need to be check.
412 ÷ 8 = 51 R 4 612 ÷ 8 = 76 R 4 512 ÷ 8 = 64 R 0 388 ÷ 8 = 48 R 4
512 is the only number divisible by 8. Thus, 78,512 is divisible by 8.

Answer: 78,512

*10. The six-digit number 8A4,B57 is divisible by 11. What is the sum of A and B?

Solution: The difference between the sums of alternating digits must be calculated.

Even-numbered digits: $8 + 4 + 5 = 17$
Odd-numbered digits: $A + B + 7$

$(A + B + 7) - 17 = A + B - 10$. This value must be divisible by 11 or equal to 0.
$A + B - 10 = -11$; $A + B = -1$ $A + B - 10 = 0$; $A + B = 10$
$A + B - 10 = 11$; $A + B = 21$
Since A and B are digits (0–9), the sum must range from 0 to $9 + 9 = 18$. 10 is the only one that fits in that range. Thus, $A + B = 10$.

Answer: 10

11. Jerry is thinking of a number between 1 and 100. The number is a multiple of 11 and is also a multiple of 5. What number is he thinking of?

Solution: If a number is divisible by 5 and 11, the number must be divisible by $5 \times 11 = 55$. The only multiple of 55 between 1 and 100 is 55.

Answer: 55

*12. If 5,A3B is a multiple of 9, what is the sum of all possible values of A + B?

Solution: Sum of digits: $5 + A + 3 + B = 8 + A + B$. This value must be equal to a multiple of 9.
$8 + A + B = 9$; $A + B = 1$
$8 + A + B = 18$; $A + B = 10$
$8 + A + B = 27$; $A + B = 19$
Since A and B are digits (0–9), the sum can be from 0 to $9 + 9 = 18$, inclusive. Thus, only 1 and 10 work. $1 + 10 = 11$.

Answer: 11

13. What is the remainder when 62,579,304 is divided by 9?

Solution: Sum of digits: $6 + 2 + 5 + 7 + 9 + 3 + 0 + 4 = 36$.
$36 \div 9 = 4$ R **0**.

Answer: 0

14. How many multiples of 3 are between 11 and 302?

Solution: $11 \div 3 = 3$ R 2 and $302 \div 3 = 100$ R 2.
Thus, the smallest multiple of 3 greater than 11 is $3 \times (3 + 1) = 3 \times 4$ because 3×3 is less than 11. The greatest multiple of 3 less than 302 is 3×100.
The numbers are $(3 \times 4, 3 \times 5, \ldots, 3 \times 100)$
From chapter 1 of counting numbers inclusive, there are $(100 - 4) + 1 = 97$ multiples.

Answer: 97 (multiples)

15. The five-digit number A5,2A1 is divisible by 3. If the five-digit number is to be as large as possible, what is the value of A?

Solution: Sum of digits: $A + 5 + 2 + A + 1 = 2A + 8$. Since, the largest digit that A could be is 9, check to see if the sum is divisible by 3 and then work down.

Let $A = 9$: $2 \times 9 + 8 = 26$; 26 is not divisible by 3
Let $A = 8$: $2 \times 8 + 8 = 24$; 24 is divisible by 3: making 85,281 the largest possible number.

Answer: 8

16. What is the smallest three-digit number that is a multiple of 5?

Solution: The smallest three digit number is 100. Since 100 ends in 0, it is a multiple of 5.

Answer: 100

17. What is largest two-digit number that is divisible by 7 and 6?

Solution: If a number is divisible by 7 and 6, the number must be divisible by $7 \times 6 = 42$. The only multiples of 42 between 1 and 100 is 42 and 84. Thus 84 is the largest two-digit number that is a multiple of 7 and 6.

Answer: 84

18. Find the remainder when 7,649,432,789 is divided by 9.

Solution: Sum of digits: 7 + 6 + 4 + 9 + 4 + 3 + 2 + 7 + 8 + 9 = 59.
59 ÷ 9 = 6 R **5**

Answer: 5

19. Marina is trying to remember her favorite number. She remembers that the number is more than 40 and less than 70 and is divisible by 8 and 7. What is her favorite number?

Solution: If a number is divisible by 8 and 7, the number must be divisible by 8 × 7 = 56. The only multiple of 56 between 40 and 70 is 56. Thus, her favorite number is 56.

Answer: 56

*20. A number is randomly chosen from 1 to 100, inclusive. What is the probability that the number is divisible by 5 and 3? Express your answer as a percent.

Solution: If a number is divisible by 5 and 3, the number must be divisible by 5 × 3 = 15. 100 ÷ 15 = 6 R 10. This means there are 6 multiples of 15 from 1 to 100, inclusive.

$$\frac{6}{100} * 100 = 6 \text{ percent}$$

Answer: 6%